Ductility Analysis of Reinforced Concrete Beam Sections under Simple Bending According to Strain Domains

Fabio H. Ferreira[1,*] and Eliomar G. Pessoa[2]

[1]*Confes Engenharia Estrutural, Sorocaba, SP, Brazil*

[2]*Federal University of Rio de Janeiro (UFRJ), Rio de Janeiro, RJ, Brazil*

Abstract: Ductility is a central requirement in the flexural design of reinforced concrete beams because it governs the capacity of a section to undergo inelastic deformation before losing resistance. The aim of this narrative review is to examine the relationship between neutral-axis depth, strain compatibility, reinforcement condition and curvature ductility in reinforced concrete sections subjected to simple bending. Peer-reviewed analytical, numerical and experimental studies were analyzed qualitatively, distinguishing conventional steel reinforcement from fiber-reinforced polymer (FRP), hybrid steel–FRP, ultra-high-performance concrete (UHPC) and steel-fiber systems. The literature shows that under-reinforced sections, in which the tensile reinforcement yields before concrete crushing, provide a more ductile response than over-reinforced sections. As the neutral axis deepens and the compression zone becomes dominant, the available deformation capacity decreases and failure becomes more brittle. Compression reinforcement, confinement, fiber-reinforced cementitious materials and hybrid steel–FRP reinforcement can modify this response, although the conventional yield-based definition of ductility must be applied with caution when non-yielding FRP reinforcement is used. Neutral-axis depth is a useful design indicator but should not be treated as an isolated or universal measure of ductility, because material properties, reinforcement ratios, confinement, geometry and the adopted definition of ultimate curvature also govern sectional behavior. The findings support a strain-compatible, mechanism-based assessment of ductility rather than reliance on reinforcement strength alone. From a sustainable-construction standpoint, these mechanisms inform durability-oriented detailing, material-efficient reinforcement selection and service-life considerations, but the reviewed studies do not establish comparative life-cycle environmental benefits.

Keywords: Reinforced concrete, Flexural ductility, Neutral axis, Strain domains, Curvature ductility, Beam sections, Moment–curvature response, FRP reinforcement. Sustainable construction, Durability, Service life, Material efficiency.

1. INTRODUCTION

Reinforced concrete beams are expected not only to resist the design bending moment but also to develop a deformation pattern that gives warning before failure. In flexure, this behavior depends on the strain compatibility between concrete and longitudinal reinforcement and on the position of the neutral axis at critical stages of loading. Curvature-based approaches are therefore widely used to describe the transition from first yielding to ultimate sectional deformation and to quantify the ductility available after yielding [1–4].

For conventional steel-reinforced sections, a favorable flexural mechanism is generally associated with yielding of the tensile reinforcement before crushing of the compressed concrete. When the tensile reinforcement ratio becomes excessive, the neutral axis moves deeper into the section, concrete compression becomes critical and the response may shift toward a less ductile failure mode [2–4]. This relationship explains why neutral-axis depth and strain domains are important design indicators, although it does not imply that the neutral-axis position alone determines ductility.

Recent research has extended this discussion beyond ordinary reinforced concrete. Compression reinforcement, confinement, ultra-high-performance concrete (UHPC), steel fibers and hybrid steel–fiber-reinforced polymer (FRP) systems can alter yielding, post-cracking behavior, ultimate curvature and energy absorption [1, 5–9]. In FRP-reinforced sections, the absence of a conventional yield plateau also requires alternative interpretations of ductility [1, 6].

Accordingly, the present study analyzes how strain domains and neutral-axis depth relate to flexural ductility in reinforced concrete beam sections under simple bending. The objectives are to organize the principal sectional mechanisms, to identify the variables that modify ductile response, and to distinguish conclusions that are well supported for conventional steel reinforcement from those that require qualification for advanced or hybrid materials.

This mechanical assessment also has implications for sustainable construction. Ductile detailing and adequate deformation capacity can support structural reliability and limit the risk of premature intervention, while appropriate reinforcement design may improve material efficiency. Corrosion-resistant reinforcement and advanced cementitious systems may reduce

*Address correspondence to this author at the Confes Engenharia Estrutural, Sorocaba, SP, Brazil; Tel: +55 11 97654-6830; E-mail: eng.fabiohf@gmail.com

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maintenance or extend service life under certain exposure conditions, although such benefits depend on their production impacts and on the complete structural design. Accordingly, this review discusses these sustainability links as design implications rather than measured environmental outcomes, since no comparative life-cycle assessment is undertaken.

1.1. Theoretical Background of Sectional Ductility

Sectional flexural analysis is based on strain compatibility and equilibrium of internal forces. As bending increases, the tensile concrete cracks, the longitudinal reinforcement carries an increasing share of the tensile force and the compression zone becomes progressively more important. The curvature at steel yielding and the curvature associated with an adopted ultimate condition define the curvature ductility, commonly expressed as the ratio between ultimate and yield curvature [2, 3]. Its value depends on the material models and on how the yield and ultimate states are defined.

Within this framework, the relative depth of the neutral axis is closely connected to the strain distribution. A shallower neutral axis corresponds to a larger tensile-steel strain for a given concrete compressive strain, favoring reinforcement yielding before concrete crushing. Analytical studies of reinforced concrete sections therefore associate lower relative neutral-axis depths with greater deformation capacity, whereas deeper neutral axes are associated with increasingly compression-controlled behavior [2–4].

This mechanism is consistent with the distinction between under-reinforced and over-reinforced flexural sections. Under-reinforced sections develop substantial tensile-steel deformation before the compressed concrete reaches its limiting strain, producing a gradual post-yield response. Conversely, excessive tensile reinforcement can move the section toward a condition in which concrete crushes with limited steel yielding, reducing warning and inelastic deformation capacity [2, 4, 10].

The response is not governed by tensile reinforcement alone. Compression reinforcement contributes to sectional strength and deformation capacity and modifies the location of the neutral axis and the moment–curvature response, while systematic procedures for calculating yield and failure curvatures show how sensitive these quantities are to the assumptions adopted [2, 3, 11]. Confinement of the compression zone can also increase the concrete strain capacity assumed at the ultimate state, thereby increasing the attainable curvature. Because these effects depend on geometry, reinforcement arrangement and constitutive assumptions, they should be interpreted through section-specific analysis rather than as fixed multipliers.

Material innovations further complicate the conventional strain-domain interpretation. FRP bars remain essentially linear-elastic up to rupture and do not exhibit the yield plateau characteristic of reinforcing steel. Consequently, a yield-to-ultimate curvature ratio cannot be transferred directly to purely FRP-reinforced members without redefining the reference deformation state. Hybrid steel–FRP reinforcement can partly restore a yielding stage through the steel component, while the FRP provides additional tensile resistance [1, 6]. UHPC and steel-fiber-reinforced cementitious materials can also modify flexural behavior through post-cracking tensile resistance, crack control and enhanced toughness [5, 7–9], but they do not eliminate the need to evaluate the strain state of the compression zone and of the longitudinal reinforcement at section level.

2. MATERIALS AND METHODS

2.1. Review Design and Source Selection

This study adopts an analytical narrative literature-review approach. Peer-reviewed analytical, numerical and experimental studies were examined for evidence concerning curvature ductility, neutral-axis depth, strain compatibility, reinforcement ratio, compression reinforcement, FRP and hybrid reinforcement, UHPC, steel-fiber reinforcement and nonlinear sectional response. Priority was given to studies that directly address the flexural behavior and ductility of reinforced concrete or related beam sections, including nonlinear analysis methods and finite-element investigations of strain evolution in concrete members [12, 13].

2.2. Interpretation Criteria

Because the selected studies use different materials, geometries, numerical models and definitions of ductility, their results were interpreted qualitatively rather than combined into a single quantitative relationship. Evidence from conventional steel-reinforced concrete was distinguished from evidence involving FRP, hybrid reinforcement, UHPC or fiber-reinforced concrete. This distinction is necessary because the presence or absence of steel yielding changes the mechanical meaning of conventional curvature-ductility indices.

The review does not generate new experimental data or propose a new design equation. Instead, it synthesizes the mechanisms supported by the literature and identifies where apparently general statements require qualification owing to differences in

reinforcement system, confinement, constitutive behavior of the materials or definition of the ultimate state. No ethical approval was required, as the study is based exclusively on published sources.

3. RESULTS AND DISCUSSION

3.1. Strain Domains, Neutral-Axis Depth and Reinforcement Condition

The reviewed studies consistently support a close relationship between strain distribution and flexural ductility. In conventional steel-reinforced concrete sections, the neutral-axis position indicates how the available concrete compressive strain is distributed between the compressed concrete and the tensile reinforcement. When the neutral axis is relatively shallow, the tensile-steel strain is larger and yielding can occur before concrete crushing, creating the deformation reserve associated with ductile flexural behavior [2–4].

This relationship is particularly useful for comparing under-reinforced and over-reinforced sections. Under-reinforced sections are designed so that the tensile reinforcement yields before the compression zone reaches its limiting condition, and the resulting post-yield curvature provides visible deformation and redistribution capacity. In over-reinforced sections, the larger compression block and deeper neutral axis increase the likelihood that the concrete reaches its ultimate compressive state before comparable yielding of the tensile steel develops, producing a more abrupt failure mechanism [2, 4, 10].

Nevertheless, the neutral-axis ratio should not be interpreted as a universal one-to-one predictor of ductility. The relationships reported in analytical formulations depend on the assumed concrete stress–strain behavior, reinforcement constitutive response, effective depth, ultimate concrete strain and definition of yield curvature [2, 3]. Concrete strength and reinforcement arrangement can change the amount of reinforcement and the section geometry required to reach a target ductility, even when similar neutral-axis ratios are obtained.

3.2. Compression Reinforcement, Confinement and Geometry

Compression reinforcement provides an additional means of modifying the sectional response. Studies on reinforced concrete beams indicate that doubly reinforced sections can sustain greater flexural demands while avoiding the unfavorable behavior associated with simply increasing the tensile reinforcement [2, 3]. Its contribution, however, must be evaluated together with the compressive behavior of the concrete and the complete reinforcement layout; it cannot be assumed to restore ductility automatically in every configuration.

Confinement and member geometry also influence the ultimate curvature that can be developed. Greater confinement of the compression zone delays the degradation of compressed concrete and increases the strain capacity available before the adopted ultimate condition is reached, while geometric proportions affect curvature and the distribution of strain over the section depth. Comparisons of moment–curvature response are therefore most meaningful when geometry, reinforcement ratio and confinement are explicitly controlled. Table **1** summarizes the sectional conditions identified in the literature and their implications for flexural ductility.

Table 1: Sectional Conditions and their Implications for Flexural Ductility

Sectional Condition	Mechanism Supported by the Literature	Implication for Ductility
Under-reinforced steel RC	Tensile steel yields before concrete reaches its limiting compressive state [2–4].	Provides a post-yield deformation reserve and a comparatively ductile flexural response.
Over-reinforced steel RC	A deeper neutral axis increases compression-controlled behavior and may lead to concrete crushing before adequate tensile-steel yielding [2, 4, 10].	Reduces deformation capacity and warning before failure.
Doubly reinforced section	Compression reinforcement modifies internal-force equilibrium and the moment–curvature response [2, 3].	Can improve flexural capacity and deformation behavior when appropriately proportioned.
Hybrid steel–FRP reinforcement	Steel provides yielding while FRP continues to contribute tensile resistance [1, 6, 14].	Allows a mixed deformation mechanism; steel-only ductility measures require qualification.
UHPC / steel-fiber systems	Post-cracking tensile resistance and toughness delay localization and alter ultimate deformation [5, 7–9].	May increase toughness and deformation capacity; performance remains material- and section-specific.

RC: reinforced concrete; FRP: fiber-reinforced polymer; UHPC: ultra-high-performance concrete. Source: prepared by the authors from the cited literature.

3.3. FRP, Hybrid and Fiber-Reinforced Systems

The evidence on FRP reinforcement demonstrates why ductility terminology must be used carefully. Pure FRP reinforcement does not yield in the same way as steel; consequently, a classical curvature-ductility ratio based on a steel yield point may be physically inappropriate. Studies therefore employ alternative deformation- or energy-based measures, while hybrid steel–FRP systems rely on the steel component to provide a recognizable yielding stage [1, 6, 14]. Finite-element investigations of ultra-high-strength concrete beams with basalt FRP and hybrid basalt FRP–steel reinforcement point in the same direction, showing that the reinforcement combination governs the post-cracking and ultimate response [15]. This difference is fundamental when ductility values are compared across reinforcement technologies.

For UHPC and fiber-reinforced concrete beams, the literature reports improved toughness and altered post-cracking behavior associated with fibers and high-performance matrices [5, 7–9]. These benefits can support larger deformation before loss of load-carrying capacity, but they should not be generalized into a single strain-domain limit applicable to ordinary reinforced concrete, since material-specific tensile behavior and fracture processes remain relevant to the interpretation of the sectional response.

3.4. Design Implications and Limitations

From a design perspective, the principal implication is that ductility should be verified through compatible strain and moment–curvature behavior rather than inferred from strength alone. Neutral-axis depth is valuable because it links reinforcement proportioning to the strain state of concrete and steel, but a robust assessment also considers the constitutive laws of the materials, compression reinforcement, confinement, geometry and the selected ultimate criterion. This broader interpretation is particularly important when non-conventional reinforcement or high-performance cementitious materials are introduced.

For green construction, a design choice that increases deformation capacity can be advantageous only when it is considered alongside resource use and long-term performance. Appropriate reinforcement proportioning may avoid unnecessary material consumption while preserving a ductile failure mechanism; conversely, reducing reinforcement solely to lower material quantities may compromise strength, crack control or durability. Hybrid steel–FRP reinforcement, UHPC and fiber-reinforced concrete may provide opportunities to balance mechanical performance with reduced maintenance or longer service life in specific applications. Those opportunities require project-specific verification of environmental exposure, material quantities, serviceability and whole-life impacts rather than extrapolation from ductility indices alone.

The available literature also has limitations. The studies reviewed include analytical formulations, numerical analyses and experiments on different types of beams, and they do not establish a common quantitative threshold that can be transferred unchanged across all systems. The strongest conclusion is therefore mechanistic: ductile flexural behavior is favored when the section can develop substantial tensile-side deformation before compression failure, whereas deeper, compression-controlled strain states reduce that deformation reserve.

The sustainability implications of this review are therefore conditional rather than quantified. The reviewed sources principally address mechanical response; they do not provide a harmonized life-cycle inventory, embodied-carbon comparison or service-life model for the reinforcement systems considered. FRP may offer corrosion-related durability benefits in certain environments, yet its production impacts, end-of-life options and non-yielding response require separate assessment. Similarly, UHPC and fiber-reinforced systems may permit improved crack control or material-efficient designs, but cement content, fiber production, structural dimensions and maintenance assumptions determine whether a net environmental benefit is achieved. Accordingly, ductility should be treated as one performance constraint within a broader sustainable-design assessment, not as a proxy for sustainability.

4. CONCLUSION

Flexural ductility in reinforced concrete beam sections is strongly associated with strain compatibility and with the position of the neutral axis. For conventional steel-reinforced concrete, under-reinforced behavior—characterized by tensile-steel yielding before concrete crushing—provides a more favorable deformation mechanism than over-reinforced, compression-controlled behavior. A deeper neutral axis generally indicates a smaller tensile-strain reserve and a greater tendency toward brittle flexural failure.

Neutral-axis depth should nevertheless not be used as an isolated universal index. Compression reinforcement, confinement, section geometry, concrete constitutive behavior, reinforcement properties and the definition of yield and ultimate curvature all modify the resulting ductility, which explains why similar neutral-

axis positions do not necessarily produce identical moment–curvature responses in different structural systems.

Advanced materials require additional care. Because FRP reinforcement does not exhibit conventional yielding, classical steel-based ductility measures may need to be replaced or supplemented by deformation- or energy-based indices. Hybrid steel–FRP systems, UHPC and fiber-reinforced concrete can improve specific aspects of deformation capacity and toughness, but their behavior remains dependent on material and sectional configuration. Overall, a mechanism-based assessment that combines strain domains, neutral-axis position and moment–curvature response provides a more defensible basis for evaluating flexural ductility than any single parameter considered in isolation. Future research should establish comparable ductility indices for steel, FRP and hybrid reinforcement and validate them against experimental moment–curvature data.

From a green-construction perspective, these findings indicate that material selection should be evaluated jointly with durability, service life, maintenance needs and material efficiency. Improved crack control or corrosion resistance may support longer-lasting members in suitable exposure conditions, while increased ductility can contribute to structural robustness and the avoidance of premature replacement. However, these potential advantages cannot be assumed to reduce environmental impacts without a consistent functional unit, exposure scenario and life-cycle assessment. Future comparative research should combine experimentally validated moment–curvature performance with service-life prediction, quantities of concrete and reinforcement, and cradle-to-grave environmental indicators.

DATA AVAILABILITY

No new datasets were generated during the present study. The data analyzed in this narrative review were obtained from the published analytical, numerical and experimental studies cited in the reference list.

AUTHORSHIP CONTRIBUTION STATEMENT

Fabio H. Ferreira: Conceptualization, Methodology, Formal analysis, Investigation, Writing – Original draft, Writing – Review & editing, Supervision, Project administration.

Eliomar G. Pessoa: Data curation, Validation, Resources, Writing – Review & editing.

LIST OF ABBREVIATIONS

BFRP	=	Basalt fiber-reinforced polymer
FRP	=	Fiber-reinforced polymer
RC	=	Reinforced concrete
SFRC	=	Steel-fiber-reinforced concrete
UHPC	=	Ultra-high-performance concrete

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

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